Faster-than-Nyquist and beyond: how to improve spectral efficiency by accepting interference

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Some of this material can be found in:

**Motivations**

**How can a BPSK reach 2 b/s/Hz?**

![Graph showing spectral efficiency (SE) vs. SNR (dB) for BPSK and pre-filtered BPSK.]


**Outline**

1. Introduction and Motivations
2. Time-Frequency Packing
3. Applications
4. Conclusions
Introduction and Motivations

Assumptions

We consider the bandlimited AWGN channel
  - Satellite channels in the absence of nonlinear amplifiers
  - Optical channels in linear regime
  - ...

We assume a multichannel/multiuser scenario (we will use the terms “channels” and “users” interchangeably)
  - Different users (e.g., different broadcasting programs)
  - Different subchannels to lower the processing speed or in case of a frequency-selective channel
**Signal model**

Transmitted signal $s(t)$

$$s(t) = \sqrt{2E_s} \sum_n \sum_\ell x_{n,\ell} p(t - nT - \tau_\ell) e^{j(2\pi \ell F t + \theta_\ell)}$$

- $E_s$: symbol energy
- $x_{n,\ell}$: $M$-ary symbol transmitted over the $\ell$-th channel during the $n$-th symbol interval
- $T$: symbol time (or time spacing)
- $F$: frequency spacing
- $p(t)$: shaping pulse
- $\theta_\ell$ and $\tau_\ell$: phase and time shifts of the $\ell$-th subcarrier
Orthogonal Signaling

- From Shannon theory, it is well known that orthogonal signaling with Gaussian inputs achieves capacity on this channel (sufficient condition but not necessary!)
- Orthogonal signaling means no ISI in case of a single-carrier transmission, no ISI and no ICI in case of a multi-carrier transmission
- For many years, digital communication systems for quasi-static channels have been designed based on orthogonal signaling

FDM (WDM)

- FDM (WDM) with Nyquist signaling:
  e.g., $p(t)$ has RRC-shaped spectrum with roll-off factor $\alpha$ and $F \geq \frac{1 + \alpha}{\tau} \ (F = \frac{1}{4} \text{ in case of OQAM, OPSK [1]})$

FDM (WDM) (cont'd)

- **FDM (WDM) with Nyquist signaling:**
  - DVB-S2 standard
  - WDM
  - Nyquist WDM [2] (interference is seen as a side effect)
  - CO-WDM [3]


OFDM:

- **OFDM:**
  - $\rho(t)$ is rectangular of duration $T$ and $F = 1/T$

- No guard bandwidth
- Efficient processing at Tx and Rx side
- On a frequency-selective channel (this is not the case of the optical ch. [4]), capacity is achieved through waterfilling

When a high-order constellation is employed, shaping [5] allows to approach the condition of "Gaussian inputs"

What is the best thing to do when low-order constellations are employed?

Change of paradigm: faster-than Nyquist (FTN) [6,7]
- The baud rate $R = 1 / T$ is increased w.r.t. Nyquist rate, thus introducing controlled ISI (as in partial response signaling)
- However, FTN does not modify the shape of the transmitted spectrum


When a high-order constellation is employed, shaping [5] allows to approach the condition of “Gaussian inputs.”

What is the best thing to do when low-order constellations are employed?

Change of paradigm: faster-than Nyquist (FTN) [6, 7]

In FTN, $T$ is selected as the smallest value giving no reduction of the minimum Euclidean distance with respect to the Nyquist case → asymptotically, the ISI-free BER performance is reached (with the optimal MAP sequence detector).

Extended to both time and frequency by Rusek and Anderson [8]: both $T$ and $F$ are reduced ⇒ both ISI and ICI arise.

Main drawback: the optimal MAP sequence detector must be used (high complexity).

Is the BER the main performance measure? In other words: are we interested in the BER performance when keeping the same code?

By the way,...CDMA:

\( F = 0 \) and a spreading code is assigned to each user. In a synchronous CDMA system \((T_\ell = T_m)\), orthogonal codes can be used to implement orthogonal signaling.

But the main success of CDMA is related to the possibility to **increment the spectral efficiency** through **overload** (in asynchronous non-orthogonal systems), thus accepting interference...
Information Theory Background

- **Information rate** or **symmetric capacity** or **i.u.d. capacity**: the mutual information $I(x; y)$ for i.u.d. symbols constrained to a finite constellation.

For a channel with finite memory, $I(x; y)$ can be computed as in [9], employing an optimal MAP symbol detector.

**Spectral efficiency** $\eta$ is our performance measure:

$$\eta = \frac{I(x; y)}{FT} \left[ \frac{\text{bit}}{\text{s} \cdot \text{Hz}} \right]$$

The receiver complexity can be taken into account by resorting to the concept of mismatched detection [10]:

if we use an optimal MAP symbol detector not for the real channel but for an auxiliary channel that approximates the original one, we obtain an achievable lower bound on $I(x; y) \Rightarrow$ we will say that the spectral efficiency depends on the employed receiver.

\[ I' (x; y) \leq I(x; y) \]

Given \( p(t) \), we optimize \( F \) and \( T \) to maximize \( \eta \) assuming different receivers.

In fact, when \( F \) and \( T \) are reduced interference increases \( \Rightarrow I(x; y) \) degrades but \( \eta \) can improve.

Example
Given $p(t)$, we optimize $F$ and $T$ to maximize $\eta$ assuming different receivers. In fact, when $F$ and $T$ are reduced interference increases $\Rightarrow I(x, y)$ degrades but $\eta$ can improve.

**Differences with FTN:**
- We use low-complexity receivers.
- We accept a degradation of the information provided the spectral efficiency is increased $\Rightarrow$ a proper code ensuring an error-free transmission in those conditions can be found.
- In other words, if we keep the same code, the performance degrades but an improvement is obtained by using a code with lower rate (higher overhead).

Receivers with different complexity can be considered.

**Single-user symbol-by-symbol detector after a proper linear front-end**

Auxiliary channel:

$$y_{k,0} = x_{k,0} h(0, 0, k) + \sum_{(n, \ell) \neq (0, 0)} x_{k-n, \ell} h(n, \ell, k) + z_k$$

ISI and ICI are assumed to be additive noise whose distribution must be optimized to improve the lower bound $h(\cdot, \cdot, \cdot)$ depends on the employed front-end (to be optimized).
Single-user receivers with complexity $O(M^L)$

$L$ interfering symbols are taken into account in the auxiliary channel model. The remaining ISI and ICI are assumed to be additive noise. The corresponding optimal MAP symbol detector has $M^L$ states.

Multi-user receivers with complexity $O(M^{LU})$

$L$ interfering symbols of the considered user and of $U$ side users are taken into account in the auxiliary channel. The remaining ISI and ICI are assumed to be additive noise. The corresponding optimal MAP symbol detector has $M^{LU}$ states.
DVB-S2 standard [11]

Spectral efficiency and practical Modcos

- $\alpha = 0.2$, $F = 1.2/T$
- Rates from 1/4 to 9/10
- Reference: PER$=10^{-7}$
- Baud rate: a few Mbaud

$p(t)$ is a RRC pulse with $\alpha = 0.2, 0.25, 0.35$
- Signals do not overlap in frequency ($F \geq (1 + \alpha)/T$)

Reference: ETSI EN 301 307 Digital Video Broadcasting (DVB); V1.1.2, 2nd generation framing structure, channel coding and modulation systems for Broadcasting, Interactive Services, News Gathering and other Broadband satellite applications, 2006.
Improvements with QPSK

Only time packing is allowed (no ICI)

- RRC, $\alpha = 0.2$
- $F = 1.2/T$
- Two possible MAP symbol detectors [12] with $L = 3$
- Further improvements with $L > 3$


Trellis processing with $L = 5$ (with QPSK, two detectors, one for each component)
**Improvements with 8-PSK**

Only time packing is allowed (no ICI)

- RRC, $\alpha = 0.2$
- $F = 1.2/T$
- Two possible MAP symbol detectors [12] with $L = 3$

**Time-Frequency Packing**

- Is a further improvement possible?
- **What happens if we also allow frequency packing**, i.e., if we optimize both $F$ and $T$?
- Limited improvement when a single-user receiver is still adopted but significant improvement is obtained when a multi-user detector is employed
- What happens if multi-user detection is not possible?
- If we filter the considered pulse, we reduce ICI and increase ISI. **But the MAP symbol detector can (partially) cope with it!**
- Similar approach adopted in [13]

Role of the Shaping Pulse

**Spectra**

- All within DVB-S2 mask
- $0.6B_0$ is the bandwidth of all pulses (where they cross -25 dB)

**Pulses after the matched filter**

- They can be implemented with a FIR of at most 15 taps (assuming Nyquist sampling)
Role of the Shaping Pulse (cont'd)

Spectral efficiency

- QPSK
- RRC with roll-off 1 provides very large SE values

Considered scheme

- LDPC encoder
- Pulse shaping
- Satellite channel
- SISO decoder
- SISO detector
- Front end filter
Practical Modcods (cont’d)

A few designed Modcods with QPSK

Modcod 1: rate-4/5 code of length 64800, $T = 0.24$

Orange Modcods: rate 1/2 from DVB-S2

Detector with complexity $O(ML)$ [14]

Improvements in DVB-S2

- FTN technique provides a gain that could enable the adoption of low-order modulation formats for higher spectral efficiency values:
  - all QPSK, 8-PSK and 16-APSK MODCODs could be replaced with MODCODs based on QPSK (more robust to non-linear effects, although PAR increases with time and frequency packing, and to synchronization errors), with a better performance

Coherent Optical Systems

- We consider coherent optical transmissions in linear regime.
- GVD and PMD can be neglected since a proper two-dimensional equalizer can perfectly cope with them [15].
- Only intentional interference generated by time and frequency packing must be considered.
- Two significant cases:
  - Linear modulator (linearized Mach-Zehnder modulator or electrically shaping the modulator driving signals through a Tx DSP with DACs).
  - Mach-Zehnder modulator.


Linear Modulator

- We assume a linear modulator and consider different pulses obtained by filtering typical RZ 33%, 50%, or 66% pulses of duration $\tau$ with a Gaussian or a 4-th order Gaussian optical filter of bandwidth $0.8/\tau$.
- Time and frequency packing.
- A single-user detector.
**Achievable spectral efficiency**

- Small differences with different pulses

![Graph of achievable spectral efficiency](image)

- **Linear Modulator**
  - **Achievable spectral efficiency**
  - Small differences with different pulses

- **Mach-Zehnder Modulator**
  - In this case, time packing is not possible (if pulses have support larger than $T$, at the modulator output we have no more a linear modulation)
  - Hence, the frequency spacing $F$ and the bandwidth $B$ of the optical filter (Gaussian or 4-th order Gaussian) have been optimized
  - Only a single-user detector has been considered
Achievable spectral efficiency

- Very limited differences with different pulses

<table>
<thead>
<tr>
<th>Parameters @ 2.5 b/s/Hz per pol.</th>
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<tbody>
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<td>FT</td>
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<td>----</td>
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<tr>
<td>QPSK</td>
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<tr>
<td>8-PSK</td>
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- To implement a 1 Tb/s superchannel and assuming sampling at 50 Gsamples/s and 2 polarizations, 5 subchannels are required for QPSK, 4 in case of 8-PSK
- **New FEC schemes must be designed** with rate much lower than those used in optical transmissions (@ 2.5 b/s/Hz: rate 0.878 for QPSK, rate 2/3 for 8-PSK)
QPSK with Gray mapping seems to be interesting, mainly for the reduced detector complexity.
ISI not coped by the receiver can require the use of an outer code and an interleaver (latency increases!)
Spectral efficiency can be considerably improved by introducing controlled interference.